Workshop "Differential Geometry, Lie Theory and Low-Dimensional Topology"

La Trobe University, Melbourne, 19-21 December, 2016

ABSTRACTS

Shintaro Akamine (Kyushu University)
Geometry of timelike minimal surfaces and null curves

Timelike surfaces in the 3-dimensional Lorentz-Minkowski space are surfaces with non-degenerate index 1 metrics. In contrast to surfaces in the 3-dimensional Euclidean space and spacelike surfaces in the Lorentz-Minkowski space, these surfaces have not always principal curvatures, that is, their shape operators are not always diagonalizable even over the complex field. For the case of timelike minimal surfaces, the problem of the diagonalizability of the shape operator is reduced to a problem of the sign of the Gaussian curvature. In this talk, we determine the sign of the Gaussian curvature of a timelike minimal surface from a viewpoint of null curves. Moreover, we also investigate the behavior of a timelike minimal surface with some kind of singularities.

Anthony Blaom (University of Auckland)
Lie algebroids and subgeometry

According to the Bonnet theorem, the metric and second fundamental form of a surface in $\mathbb{R}^3$ completely characterise the surface, up to isometry. We explain how Lie algebroids arise naturally in this old problem and present a new approach to proving Bonnet-type theorems in the general setting of Klein geometries (conformal geometry, projective geometry, CR geometry, etc). Our approach is based on an infinitesimal characterization of smooth maps into a homogeneous space which generalises Èlie Cartan's characterization of the smooth maps into a Lie group. No familiarity with Lie algebroids will be assumed.

Philip Broadbridge (La Trobe University)
Exact solutions for geometric evolution of metal surfaces.

It is outlined how analytic solutions have been obtained for (i) grain boundary evolution by the curve shortening equation, (ii) grain boundary evolution by curvature-driven surface diffusion, and more recently (iii) a cone with prescribed angle, evolving by mean curvature "surface shortening equation". The latter is ongoing, jointly with Gallage, Triadis and Cesana.

Owen Dearricott (The University of Melbourne)

Positive curvature in dimension 7

We discuss a few spaces that carry positive sectional curvature in dimension 7 that relate to 3-Sasakian geometry.

Keegan Flood (The University of Auckland)

Metric projective geometries and projective compactification

Two affine connections on a manifold are projectively equivalent if they have the same geodesics up to reparametrization. A projective manifold is a smooth manifold equipped with a class of projectively equivalent torsion-free affine connections. Given a projective manifold it is natural to ask whether there is a connection in its projective class arising as the Levi-Civita connection of a metric. The answer is related to the existence of a solution to a geometric PDE known as the metrizability equation. We show that under certain assumptions the degeneracy locus of a solution to the metrizability equation is a smoothly embedded separating hypersurface endowed with either a projective or a conformal structure. In doing so we relate scalar curvature conditions on the interior of a manifold with boundary to the manner in which the projective structure of the interior extends to the boundary. This is joint work with A. R. Gover.

Wolfgang Globke (The University of Adelaide)

Compact pseudo-Riemannian Einstein manifolds in low dimensions

Let M be pseudo-Riemannian homogeneous Einstein manifold of finite volume, and suppose a connected Lie group G acts transitively and isometrically on M. We study such spaces M in the two cases where G is solvable or semisimple. In the solvable case, M is compact and $M = G/\Gamma$ for a lattice Γ in G. Solvable Lie algebras $\mathfrak{g}$ with invariant Einstein scalar product exist only for $\dim \mathfrak{g} \leq 7$ and Witt index ≤ 2. The existence of a compact M then requires the existence of a lattice in the solvable group G, and in dimensions ≤ 7, this requires G to be nilpotent. We conjecture that this is true for any dimension. In fact, this holds if Schanuel's Conjecture on transcendental numbers is true. In the case where G is semisimple, M splits as a pseudo-Riemannian product of Einstein quotients for the compact and the non-compact factors of G. This is joint work with Yuri Nikolayevsky.

Rod Gover (University of Auckland)

Nearly Kaehler geometry, (2,3,5) distributions, and projective differential geometry.

Nearly Kaehler geometries are one of the most important classes in the celebrated Gray-Hervella classification of almost Hermitian geometries. A Cartan (2, 3, 5)-distribution is the geometry arising from a maximally nondegenerate distribution of 2-planes in the tangent bundle of a 5-manifold. We reveal a beautiful picture of how any (2, 3, 5)-geometry arises as the induced geometry on the boundary at infinity of a nearly Kaehler manifold; included is an explanation of how the almost complex structure of the nearly Kaehler geometry degenerates at the boundary to yield there the distribution generating the (2, 3, 5) structure. This uses the algebraic structure of the imaginary (split) octonions, and also new results and ideas from the general theory of Cartan holonomy reduction (as applied to projective geometry).

This is based on joint work with Roberto Panai and Travis Willse.

Takahiro Hashinaga (National Institute of Technology Kitakyushu College)

On homogeneous Lagrangian submanifolds in complex hyperbolic spaces

In this talk, we study homogeneous Lagrangian submanifolds in complex hyperbolic spaces. We show there exists a correspondence between compact homogeneous Lagrangian submanifolds in complex hyperbolic spaces and ones in complex Euclidean spaces (or equivalently, complex projective spaces). We also introduce classification results of non-compact homogeneous Lagrangian submanifolds in complex hyperbolic spaces obtained by actions of connected closed subgroups of the solvable part of the Iwasawa decomposition. This talk is based on a joint work with Toru Kajigaya (OCAMI, Japan).

Yuta Hatakeyama (Kyushu University)

Approximation of the Curvature and the Torsion of Curves by the Discrete Curvature and the Discrete Torsion

The shape of a smooth curve in R^3 is determined uniquely by the curvature and the torsion. On practical applications, very often we can get information only about discrete points on a given curve. Hence, it is important to approximate the curvature and the torsion by data of only discrete points and to evaluate the errors. In this talk, we discretize a smooth curve by choosing discrete points on the original curve and define the discrete curvature and the discrete torsion at these points. We evaluate the errors between the curvature and the discrete curvature, and the torsion and the discrete torsion when we increase the number of discrete points on the smooth curve. We also apply our results to some explicit examples.

Joshua Howie, Jessica Purcell (Monash University)

Quasi-Fuchsian checkerboard surfaces

Weakly generalised alternating knots and links were introduced in the speaker's PhD thesis. Links in this class have alternating diagrams on closed orientable surfaces in the 3-sphere. Several geometric properties of the complements of planar alternating knots carry over to this new class. In particular, we show that in many cases where the knot complements are hyperbolic, both the associated checkerboard surfaces are quasi-Fuchsian.

Isami Koga and Yasuyuki Nagatomo (Kyushu University)

Equivariant holomorphic embeddings from the complex projective line into a complex Grassmannian of 2-planes

In this talk, we study a classification problem of $SU(2)$-equivariant holomorphic embedding of complex projective line $\mathbb{C}P^1$ into a complex Grassmannian of 2-planes $Gr_{n-2}(\mathbb{C}^n)$. In order to classify such maps, we study holomorphic vector bundles over $\mathbb{C}P^1$ of rank 2 with $SU(2)$-action, invariant metric and invariant connection. We classify such bundles and consequently we obtain the moduli space of $SU(2)$-equivariant holomorphic maps.

Miyuki Koiso (Kyushu University)
Stability and bifurcation for surfaces with constant mean curvature
A surface with constant mean curvature (CMC surface) is an equilibrium surface of the area functional among surfaces which enclose the same volume (and satisfy given boundary conditions). A CMC surface is said to be stable if the second variation of the area is non-negative for all volume-preserving variations. In this talk we first give criteria for stability of CMC surfaces in the three-dimensional euclidean space. We also give a sufficient condition for the existence of smooth bifurcation branches of fixed boundary CMC surfaces, and we discuss stability/instability issues for the surfaces in bifurcating branches. By applying our theory, we determine the stability/instability of some explicit examples of CMC surfaces.

James McCoy (University of Wollongong)
Curvature contraction flow of axially symmetric hypersurfaces
We show that axially symmetric hypersurfaces contracting under a class of fully nonlinear curvature flow in either Euclidean space or in the sphere converge to round points. The result in Euclidean space is joint work with F. Mofarreh and V.-M. Wheeler.

Margaret McIntyre (University of Ghana)
A missing link in the classification of immersed curves which extend to immersed surfaces?
Blank studied the problem of extending a normal immersed circle f in the plane to the immersion of a closed orientable surface with boundary f, by defining combinatorial structures (subwords) in a word assigned to f. In his PhD thesis on the classification of immersions which are bounded by curves in surfaces, [2010, Technischen Universitat Darmstadt] Dennis Frisch replaced the plane with $\mathbb{S}^2$ and introduced a new class of subword which could account for extension to a surjective immersion. We provide an example and demonstrate Frisch's solution. Then using the same immersed circle f, we demonstrate another combinatorial structure in the word assigned to f, the structure of linked negative groups. The existence of such linked negative groups reopens the problem of finding all possible extensions to a normal immersed curve in a surface.

Marcel Nicolau (Universitat Autonoma de Barcelona)
Foliations and webs inducing Galois coverings
This talk is based on the joint paper (*A. Beltran, M. Falla Luza, D. Marin, M. Nicolau, Foliations and webs inducing Galois coverings, International Mathematics Research Notices, 12 (2016) 3768–3827*) and on current joint work with D. Marin.

A k-web is a geometric structure defined locally as the superposition of k foliations. The image of a holomorphic foliation of degree k on the complex projective space $\mathbb{P}^n$ by its Gauss map is a k-web. Motivated by previous work of Cerveau and Deserti, we introduce the notion of Galois holomorphic foliations on $\mathbb{P}^n$ as those whose Gauss map is a Galois covering when restricted to an appropriate Zariski open subset. This definition can be extended in a natural way to the notion of Galois k-webs defined on arbitrary projective manifolds. We characterize Galois foliations on $\mathbb{P}^2$ in terms of geometric data: their inflection divisor and their singularities. In particular we show that the classification of homogeneous Galois foliations corresponds to Klein's classification of ramified coverings of the projective line $\mathbb{P}^1$. We also determine the homogeneous Galois foliations on $\mathbb{P}^2$ that are flat.

Yurii Nikonorov (Southern Mathematical Institute VSC RAS)

The evolution of positively curved invariant metrics on the Wallach spaces under the Ricci flow

This talk is based on a recent joint paper with N.A. Abiev (*N.A. Abiev, Yu.G. Nikonorov, The evolution of positively curved invariant Riemannian metrics on the Wallach spaces under the Ricci flow, Annals of Global Analysis and Geometry, 50:1 (2016), 65–84*). We studied the evolution of positively curved metrics on the Wallach spaces $SU(3)/T_{\max}$, $Sp(3)/Sp(1) \times Sp(1) \times Sp(1)$, and $F_4/Spin(8)$. We proved that for all Wallach spaces, the Ricci flow evolves all generic invariant Riemannian metrics with positive sectional curvature into metrics with mixed sectional curvature. Moreover, we proved that for the spaces $Sp(3)/Sp(1) \times Sp(1) \times Sp(1)$ and $F_4/Spin(8)$, the Ricci flow evolves all generic invariant Riemannian metrics with positive Ricci curvature into metrics with mixed Ricci curvature. We also obtained similar results for some more general homogeneous spaces.

Osamu Saeki (Kyushu University)

Indefinite fibrations on 4-manifolds

A broken Lefschetz fibration (BLF, for short) is a smooth map of a closed oriented 4-manifold onto a closed surface whose singularities consist of Lefschetz critical points together with indefinite folds (or round singularities). Such a class of maps was first introduced by Auroux-Donaldson-Katzarkov in relation to near-symplectic structures. In this talk, we give a set of explicit moves for indefinite fibrations, which include BLFs and indefinite generic maps, and give an elementary and constructive proof to the fact that any map into the 2-sphere is homotopic to a BLF with embedded round image. We also show how to realize any given null-homologous 1-dimensional submanifold with prescribed local models for its components as the round locus of a BLF. These algorithms allow us to give a purely topological and constructive proof of a theorem of Auroux-Donaldson-Katzarkov on the existence of broken Lefschetz pencils with embedded round image on near-symplectic 4-manifolds. This is a joint work with R. İnanç Baykur (University of Massachusetts).

Hiroshi Tamaru (Hiroshima University)

Left-invariant metrics and submanifold geometry

For left-invariant metrics on Lie groups, we can construct submanifolds in the spaces of all left-invariant metrics. We expect that nice left-invariant metrics are corresponding to nice submanifolds. In this talk, we introduce our framework, and mention some results related to such correspondence.